Relating Calculus and Physics via Falling Bodies

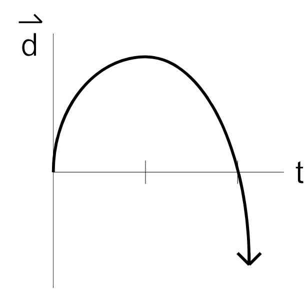
Calculus can be thought of as elementary mathematics (algebra, geometry, trigonometry) enhanced by the limit process. Calculus was independently invented by two men in the late 1600's. Sir Isaac Newton was an Englishman and arguably the greatest physicist of all time. He invented *fluxions*, a version of calculus, around 1665 but did not announce his work until 1687. Gottfried Liebnez, a German mathematician who was unaware of Newton's unpublished work, began independently developing calculus in 1673. Liebnez's notations of *dx* for differentiation and $\int$ for integration eventually became the standard. Newton's method of indicating derivatives with dots over variables ($\dot{x}$, $\ddot{x}$) is similar to today's use of primes (x', x'').

Falling body graphs and equations illustrate how calculus relates to the physics of motion:

DISPLACEMENT

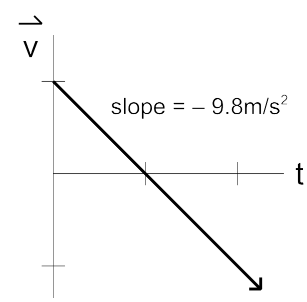
PHYSICS EQUATION: $d = v_i t + \frac{1}{2} a t^2$

CALCULUS EQUIVALENT: $y_f = y_i + v_i t + \frac{a t^2}{2}$



VELOCITY

PHYSICS EQUATION: $v_f = v_i + at$



PHYSICS GRAPH CONCEPT:
Velocity is the slope of the displacement versus time graph.

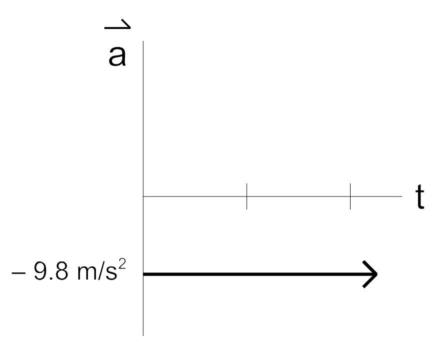
CALCULUS EQUIVALENT:
Velocity is the first derivative of displacement with respect to time.
The first derivative of a parabolic function is a linear function.

$$v_f = \frac{dy_f}{dt} = \frac{d}{dt}(y_i + v_i t + \frac{at^2}{2}) = \frac{d}{dt}(y_i) + \frac{d}{dt}(v_i t) + \frac{d}{dt}(\frac{at^2}{2}) = 0 + v_i + 2(\frac{a}{2})t = v_i + at$$

ACCELERATION

PHYSICS EQUATION:

$$a = -9.8\ m/s^2$$



PHYSICS GRAPH CONCEPT:
Acceleration is the slope of the velocity versus time graph.

CALCULUS EQUIVALENT:
Acceleration is the second derivative of displacement with respect to time, or the first derivative of velocity with respect to time. The second derivative of a parabolic function is a constant; the first derivative of a linear function is a constant.

$$a = \frac{d^2 y_f}{dt^2} = \frac{dv_f}{dt} = \frac{d}{dt}(v_i + at) = \frac{d}{dt}(v_i) + \frac{d}{dt}(at) = 0 + a = a$$