12 Rotation

Reading: Angular Momentum

Name____________________________________

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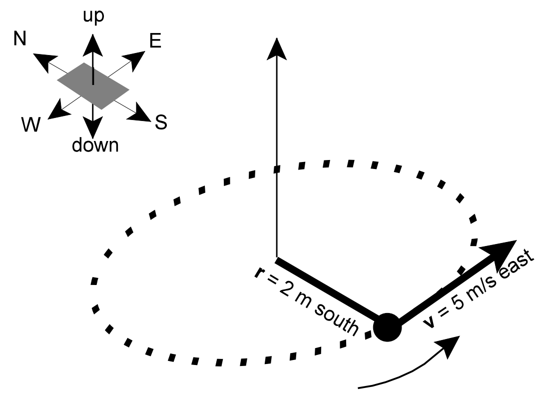
You have already learned about linear momentum, the product of an object's mass and its linear velocity. You know that linear momentum is conserved in the absence of any external unbalanced forces. There is an analogous concept for rotational motion. A rotating body possesses **angular momentum** which is conserved in the absence of any external unbalanced **torques**. The angular momentum (designated as **L**) of a point on a body is the vector cross-product of its distance from the axis of rotation and its linear momentum: $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$

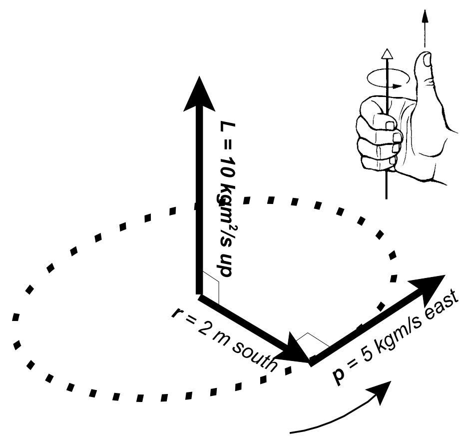
To understand what is meant by "vector cross-product", consider a 1 kg ball spun in a horizontal circle on a 2 m long string with a tangential velocity of 5 m/s. The ball's linear momentum **p** when its velocity was eastward would be:

$$\boldsymbol{p} = m\,\boldsymbol{v} = (1\ kg)(5\ m/s\ east) = 5\ kg{\cdot}m/s\ east$$



As shown in the diagram, the ball also has a **radius vector** of 2 meters south when its velocity is eastward. The angular momentum, the cross-product of the radius and linear momentum vectors, would thus have magnitude of 2 m × 5 kg·m/s = 10 kg·m^2/s.

Since angular momentum is a vector, we must somehow assign its direction. It is supposed to be perpendicular to both **r** and **p**, so if **r** and **p** lie in the x-y plane, the cross-product **L** lies along the z-axis. Its orientation is found by the **right-hand rule**. This rule says to extend the fingers and thumb of your right hand. Then position your hand so that as you curl your fingers they first point in the direction of the first vector (**r**) and then sweep toward the direction of the second vector (**p**). Your thumb will then point in the direction of the cross-product (**L**). The diagram at right shows that the cross-product of a south **r** vector with an east **p** vector would be an **L** vector that points upward.



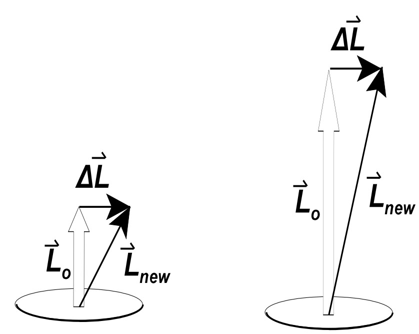
Thus we arrive at the angular momentum of the ball:

$$\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p} = 2\ m\ south \times 5\ \frac{kg{\cdot}m}{s}\ east = 10\ \frac{kg{\cdot}m^2}{s}\ upward$$

The reason for identifying the angular momentum is that it cannot change unless there is an external torque. Suppose we pull inward on the string holding the ball. Since our pull has no component perpendicular to the radius vector, it exerts no torque and the ball's angular momentum cannot change. If the ball is pulled inward 1 meter, its radius vector has been halved. Thus its linear momentum must double to maintain the same angular momentum (2 × 5 = 1 × 10). The only way the linear momentum of the ball can double is if its speed doubles.

You have probably seen a similar effect in ice-skating. Skaters often extend one arm and leg and go into a spin. As they draw their extended limbs inward to their body, their spin speed rises dramatically. This occurs because there is no torque on the skater during the spin and angular momentum is thus conserved. Ballet dancers also use this physics property in a movement called the *pirouette*.

The conservation of angular momentum explains why objects are more stable when spun, from the flight of Frisbees to spinning satellites launched from the space shuttle. When an object spins with a high velocity, it has a large angular momentum vector aligned with the axis of rotation. An applied torque will cause a certain change in the angular momentum (Δ**L**). The diagram at right shows how the torque will tilt the object less when the object's original angular momentum ($\mathbf{L}_o$) is larger.



An external torque adds ΔL to the existing angular momentum L_o, forming the tilted L_{new}.

The larger L_o is, the less L_{new} is tilted by the external torque's ΔL.